

University of California at Berkeley  
Physics 129A  
Professor Freedman  
Fall 2004  
October 2, 2004  
Homework #5 (Due: Monday October 11)

1. Mass Differences of Mirror Nuclei.

The semi-empirical mass formula was devised to estimate the mass of the most bound nucleus for a particular  $A$ . In this problem we will try to make estimates inspired by this relationship to make estimates of mass differences for mirror nuclei (nuclei with the same  $A$  but different values of  $N$  and  $Z$  within isospin multiplets). This is beyond the scope of the original mass formula and you will have to think carefully to understand why using this relationship is justified.

(a) Choose the terms in the semi-empirical mass relation that would be appropriate for estimating the differences in the ground state masses of the mirror pairs: (i)  $^{13}\text{C}$  and  $^{13}\text{N}$  (ii)  $^{17}\text{O}$  and  $^{17}\text{F}$  (Justify your choices). Using the coefficients given in class estimate the mass differences for these two pairs and compare your results to the measured masses in the literature. (Note: we want to compare the nuclear masses here, the literature often reports atomic masses.)

(b) The lowest lying isospin-1 multiplet in the  $^{12}\text{B}$ - $^{12}\text{C}$ - $^{12}\text{N}$  triplet of nuclei was discussed in class. This multiplet includes the ground states of  $^{12}\text{B}$  and  $^{12}\text{N}$  and an excited state of  $^{12}\text{N}$ . What terms in the semi-empirical mass relation would be appropriate for estimating the differences in the mass of the members of this isotriplet. Make a numerical estimate and compare your result to the measured masses. (Remember in the case of  $^{12}\text{C}$  we are interested in the mass of the nucleus in an excited state.)

2. Nuclear Energy Levels and Alpha decay.

Some unstable nuclei alpha decay to excited energy levels of the daughter nucleus. The daughter then emit one or more gamma rays as it decays to its ground state. Imagine you have a sample of these unstable nuclei. First, you measure the following peaks in the spectrum of alpha particle energies: 5.137, 5.173, 5.208, 5.338, 5.421 MeV. You then measure the following peaks in the spectrum of gamma rays: 0.084, 0.133, 0.169, 0.215, 0.217 MeV

Using this data, construct the energy level diagram of the daughter nucleus, showing the appropriate alpha and gamma decays.

You can check your answer by trying to figure the nucleus by searching the appropriate web site.

[Extra part for the curious, but not for any credit: Can you explain why particular gamma transitions occur, while others do not?]

### 3) Muon Spallation Background (Or "Helping your GSI with his research")

This problem is one we have to deal with in the KamLAND experiment. In neutrino and dark matter experiments, one is trying to detect very rare particle interactions, so any possible radioactive background can cause problems. For this reason, most of these experiments are very clean, as well as deep underground to shield them from cosmic ray muons.

Even though KamLAND is 1 km underground, some of the very high-energy muons still reach the detector (about one muon every 3 seconds). The muons create such a high energy signal in the detector ( $\sim$ GeV) that we have no problem separating them from the neutrino signal we are interested in ( $\sim$ MeV). But, these very high-energy muons can destroy stable nuclei by knocking nucleons out of the nucleus. This is commonly referred to as muon spallation. For this problem, you can think of KamLAND as basically a huge tank of mineral oil (carbon and hydrogen). The muon can turn  $^{12}\text{C}$  into any nuclei of equal or less neutrons and protons.

a) Using the Segre chart at the NNDC ([www.nndc.bnl.gov](http://www.nndc.bnl.gov)), or in a table of isotopes, find all the possible unstable nuclei that could be produced by muon spallation. (Using the fact that the KamLAND detector is blind for about 150 $\mu\text{s}$  after a muon event, ignore anything that decays on a much shorter timescale.) List the isotope, half-life, and decay modes. (On the NNDC website, you can get this information by entering the isotope in the box at the right and the information will be displayed below the Segre chart. The decay mode shorthand is: EC=electron capture, B=beta, A=alpha, P=proton, N=neutron)

b) All of these backgrounds would make the neutrino experiment impossible, except we are lucky. In KamLAND, each reactor neutrino interaction creates a neutron. This neutron bounces around in the detector for about 200ms and then interacts with a hydrogen nucleus. We identify neutrino events by this pair of interactions (neutrino/neutron.) So of the radioactive elements you found in part a, how many of them also produce a neutron? These remaining isotopes are the troublesome backgrounds for the KamLAND reactor neutrino experiment.

[Extra part for the curious, but not for credit: Can you guess how KamLAND deals with these remaining backgrounds?]